F12T1 : 3. Schulaufgabe Mathematik am 28.04.16

1.0 $f(x) = \dfrac{2 + 2\ln(x^2)}{x^2}$; $D = \mathbb{R} \setminus \{0\}$

1.1 $f(-x) = \dfrac{2 + 2\ln((-x)^2)}{(-x)^2} = \dfrac{2 + \ln(x^2)}{x^2} = f(x)$ } A-Sym zur y-Achse

D sym

$f(x) = 0 \Rightarrow 2 + 2\ln(x^2) = 0 \Leftrightarrow \ln(x^2) = -1$

$\Leftrightarrow x^2 = \frac{1}{e} = e^{-1} \Leftrightarrow x_{1/2} = \pm\frac{1}{\sqrt{e}} = \pm e^{-1/2} \quad (\approx 0{,}61)$

1.2 $x \to \infty : f(x) \to \text{"}\frac{2 + 2\cdot\infty}{\infty}\text{"} \overset{L.H}{\rightsquigarrow} \frac{2 \cdot \frac{1}{x^2} \cdot 2x}{2x} = \frac{2}{x^2} \to 0$

$x \to 0^+ : f(x) \to \frac{\text{"}2 - 2\cdot\infty\text{"}}{0} \to -\infty$

$x \to -\infty : f(x) \to 0$

$x \to 0^- : f(x) \to -\infty$ } wg. sym.

1.3 $f'(x) = \dfrac{x^2 \cdot 2 \cdot \frac{1}{x^2} \cdot 2x - (2 + 2\ln(x^2)) \cdot 2x}{x^4} =$

$= \dfrac{4x - 4x - 4x\ln(x^2)}{x^4} = \dfrac{-4 \cdot \ln(x^2)}{x^3}$

$f''(x) = \dfrac{-x^3 \cdot 4 \cdot \frac{1}{x^2} \cdot 2x + 4\ln(x^2) \cdot 3x^2}{x^6} =$

$= \dfrac{-8x^2 + 12x^2\ln(x^2)}{x^6} = \dfrac{-8 + 12\ln(x^2)}{x^4}$

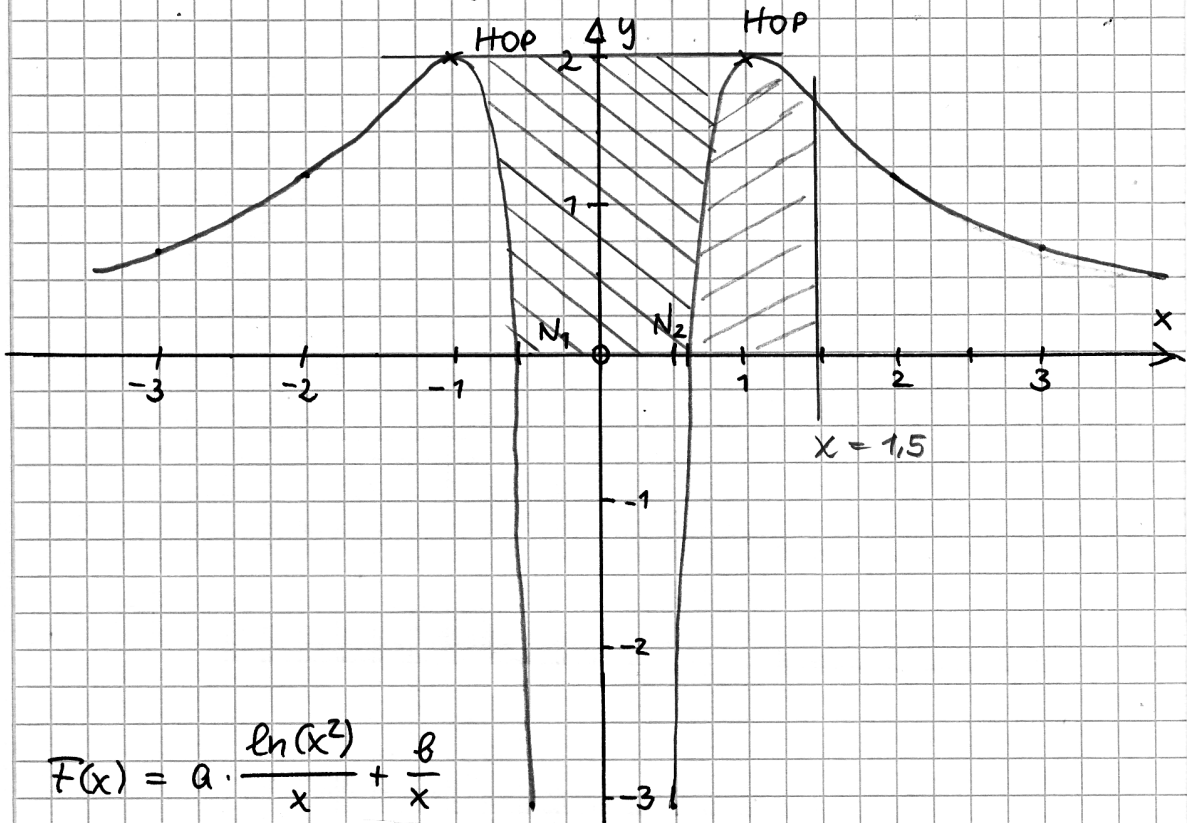
1.4 $f'(x) = 0 \Rightarrow 4\ln(x^2) = 0 \Leftrightarrow \ln(x^2) = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x_{1/2} = \pm 1$

$f''(1) = \dfrac{-8 + 12 \cdot \ln(1)}{1^4} = -8 < 0$; $f(1) = 2 \Rightarrow HOP_1(1|2)$

$HOP_2(-1|2)$ wg. sym.

F12T1 3. Schulaufgabe am 28.04.16

1.5



1.6.0 $F(x) = a \cdot \frac{\ln(x^2)}{x} + \frac{b}{x}$

1.6.1 $F'(x) = a \cdot \frac{x \cdot \frac{1}{x^2} \cdot 2x - \ln(x^2) \cdot 1}{x^2} - \frac{b}{x^2} =$

$= \frac{2a - a \cdot \ln(x^2) - b}{x^2} = \frac{2a - b - a\ln(x^2)}{x^2}$

$-a = 2 \Leftrightarrow a = -2$; $2a - b = 2 \Leftrightarrow -4 - b = 2 \Leftrightarrow b = -6$

1.6.2 An der Stelle $x_1 = \frac{1}{\sqrt{e}}$ einen TIP, weil dort $F'(x) = f(x)$ eine NST m. VZW von Minus nach Plus hat.

1.6.3 $\int_{1/\sqrt{e}}^{k} f(x)\,dx = \left[\frac{-2\ln(x^2) - 6}{x}\right]_{1/\sqrt{e}}^{k} =$

$= \frac{-2\ln(k^2) - 6}{k} - \left(\frac{-2 \cdot \ln((e^{-1/2})^2) - 6}{1/\sqrt{e}}\right) =$

$= \frac{-2 \cdot 2\ln(k) - 6}{k} + \frac{2 \cdot (-1) + 6}{1/\sqrt{e}} = 4\sqrt{e} - \frac{4\ln(k) + 6}{k}$

1.6.4 $A_2 = A_{\square} - 2 \cdot A(1) = 2 \cdot 2 - 2\left(4\sqrt{e} - \frac{4\ln(1) + 6}{1}\right)$

$= 4 - 8\sqrt{e} + 12 = 16 - 8\sqrt{e} \quad (\approx 2{,}81)$

F12T1 3. Schulaufgabe Mathematik am 28.04.16

2.0 $A(3|-4|3)$; $B(-5|0|7)$; $C(-1|-2|-3)$

$E_k: x_1 + 2x_2 + \frac{1}{k}x_3 = 12$; $k \in \mathbb{R} \setminus \{0\}$

2.1 $\vec{AB} = \begin{pmatrix} -5-3 \\ 0+4 \\ 7-3 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$; $\vec{AC} = \begin{pmatrix} -1-3 \\ -2+4 \\ -3-3 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix}$

$\vec{AB} \times \vec{AC} = \begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix} \times \begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix} = \begin{pmatrix} -24-8 \\ -16+48 \\ -16+16 \end{pmatrix} = \begin{pmatrix} -32 \\ -64 \\ 0 \end{pmatrix} = -32 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

$F: \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \circ \begin{pmatrix} x_1+5 \\ x_2-0 \\ x_3-7 \end{pmatrix} = 0 \Rightarrow F: x_1 + 2x_2 + 5 = 0$

$F: \perp x_1 - x_2$-Ebene

2.2 $E_k: x_1 + 2x_2 + \frac{1}{k}x_3 = 12$ (I)

$F: x_1 + 2x_2 = -5$ (II)

$I - II: \frac{1}{k}x_3 = 17 \Leftrightarrow x_3 = 17k$

$II: x_1 + 2x_2 = -5 \Leftrightarrow x_1 = -5 - 2x_2 = -5 - 2\lambda$; $x_2 = \lambda$

$x_1 = -5 - 2\lambda$
$x_2 = 0 + 1\lambda$ $\Rightarrow S: \vec{x} = \begin{pmatrix} -5 \\ 0 \\ 17k \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$
$x_3 = 17k + 0\lambda$

2.3 $\vec{n}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; $\vec{n}_F = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$; $\vec{n}_1 \circ \vec{n}_F = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = 1 + 4 = 5$

$\cos(\beta) = \frac{5}{\sqrt{1+4+1} \cdot \sqrt{1+4}} = \frac{5}{\sqrt{6} \cdot \sqrt{5}} = \frac{5}{\sqrt{30}} \Rightarrow \beta = 24{,}09°$

2.4 $e: \vec{x} = \tau \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ in $F: 1 \cdot 1\lambda + 2 \cdot 2\lambda = -5 \Leftrightarrow 5\lambda = -5$

$\lambda = -1$ in $e: \vec{L} = -\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \Rightarrow L(-1|-2|0)$